

# Thermomechanical response of generalized thermoelastic diffusion with one relaxation time due to time harmonic sources

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Received 18 September 2006; received in revised form 17 February 2007; accepted 19 February 2007

Available online 3 April 2007

## Abstract

A general solution to the field equations of generalized thermodiffusion in an elastic solid has been obtained, in the transformed form, using the Fourier transform. Assuming the disturbances to be harmonically time dependent, the transformed solution is obtained in the frequency domain. As an application, concentrated and distributed sources have taken to illustrate the utility of the approach. The transformed solutions are inverted numerically, using a numerical inversion technique to invert the Fourier transform. The variations of concentration distribution, chemical potential distribution and effect of diffusion on the normal stress and temperature distribution have been depicted graphically for Lord and Shulman (L–S) and coupled thermoelastic (C–T) theories of thermoelasticity.

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*Keywords:* Generalized thermoelastic diffusion; Time harmonic; Concentrated and distributed loads; Fourier transform

## 1. Introduction

Boit [1] formulated the theory of coupled thermoelasticity to eliminate the paradox inherent in the classical uncoupled theory that elastic changes have no effect on temperature. The heat equations for both theories are diffusion type predicting infinite speeds of propagation for heat waves contrary to physical observations. Lord and Shulman [2] introduced the theory of generalized thermoelasticity with one relaxation time by postulating a new law of heat conduction to replace the classical Fourier's law. This law contains the heat flux vector as well as its time derivative. It contains also new constant that acts as relaxation time. The heat equation of this theory is of the wave type, ensuring finite speeds of propagation for heat and elastic waves. The remaining governing equations for this theory, namely, the equations of motion and constitutive relations remain the same as those for the coupled and uncoupled theories. The theory was extended by Dhaliwal and Sherief [3] to general anisotropic media in the presence of heat sources.

The study of diffusion phenomenon is of great deal of interest due to its many applications in geophysics and industrial applications. Diffusion can be defined as the random walk, of an ensemble of particles, from regions of high concentration to regions of lower concentration. In integrated circuit fabrication, diffusion is used to introduce “dopants” in controlled amounts into the semiconductor substrate. In particular, diffusion is used to form the base and emitter in bipolar transistors, form integrated resistors, form the source/drain regions in Metal Oxide Semiconductor (MOS) transistors and dope poly-silicon gates in MOS transistors. Study of the phenomenon of diffusion is used to improve the conditions of oil extractions (seeking ways of more efficiently recovering oil from oil deposits). These days, oil companies are interested in the process of thermoelastic diffusion for more efficient extraction of oil from oil deposits.

Nowacki [4–7] developed the theory of thermoelastic diffusion. In this theory, the coupled thermoelastic model is used. Genin and Xu [8] investigated a problem on thermoelastic plastic metals with mass diffusion.

Sherief et al. [9] developed the generalized theory of thermoelastic diffusion with one relaxation time, which allows the finite speeds of propagation of waves. Olesiak and Pyryev [10]

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### Nomenclature

$\theta = T - T_0$ , $T$	absolute temperature
$P$	chemical potential per unit mass
$\beta_1 = (3\lambda + 2\mu)\alpha_t$ , $\alpha_t$	coefficient of linear thermal expansion
$\beta_2 = (3\lambda + 2\mu)\alpha_c$ , $\alpha_c$	coefficient of linear diffusion expansion
$K$	coefficient of thermal conductivity
$a$	coefficients describing the measure of thermoelastic diffusion effects
$b$	coefficients describing the measure of diffusive effects
$\sigma_{ij}$	components of stress tensor
$e_{ij}$	components of strain tensor
$C$	concentration distribution
$e = e_{kk}$	cubic dilatation
$\rho$	density
$\tau$	diffusion relaxation time

$u_i$	displacement vector
$\delta_{ij}$	Kronecker's delta
$\lambda, \mu$	Lame's constants
$C_E$	specific heat at constant strain
$T_0$	temperature of the medium in its natural state assumed to be such that $ \theta/T_0  < 1$
$\tau_0$	thermal relaxation time
$D$	thermoelastic diffusion constant

and

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}); \quad i, j = 1, 2, 3$$

$$\nabla = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

discussed a coupled quasi-stationary problem of thermodiffusion for an elastic cylinder. Sherief and Saleh [11] investigated the problem of a thermoelastic half-space in the context of the theory of generalized thermoelastic diffusion with one relaxation time. Singh [12,13] discussed the reflection phenomena of waves from free surface of an elastic solid with generalized thermodiffusion. Recently, Aouadi [14] studied the thermoelastic–diffusion interactions in an infinitely long solid cylinder subjected to thermal shock on its surface with a permeating substance. Aouadi [15] investigated the problem of thermoelastic half-space with a permeating substance in contact with the bounding plane in context of the theory of generalized thermoelastic diffusion with one relaxation time and with variable electrical and thermal conductivity.

The formulation and solution of the problems in frequency domain are simpler than in the time-domain. This is, off course, due to the absence of the time variable in the frequency domain formulation and hence, the transformation of the dynamic problem into the static like problem. Many researchers have dealt with the dynamic problems in the frequency domain. Sharma et al. [16] discussed time harmonic sources in a generalized thermoelastic continuum. Kumar and Rani [17] investigated the dynamic response of a homogeneous, isotropic thermoelastic half-space with voids subjected to time harmonic mechanical and thermal sources.

The present investigation is to determine the components of displacement, stress, temperature distribution and chemical potential distribution in an isotropic homogeneous elastic solid with generalized thermoelastic diffusion subjected to concentrated and distributed loads.

## 2. Basic equations

Following, Sherief et al. [9], the governing equations for an isotropic, homogeneous elastic solid with generalized ther-

moelastic diffusion with one relaxation time in the absence of body forces and heat sources are:

The constitutive equations

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}[\lambda e_{kk} - \beta_1 \theta - \beta_2 C] \quad (1)$$

$$P = -\beta_2 e_{kk} + bC - a\theta \quad (2)$$

The equation of motion

$$\mu u_{i,jj} + (\lambda + \mu) u_{j,ij} - \beta_1 \theta_{,i} - \beta_2 C_{,i} = \rho \frac{\partial^2 u_i}{\partial t^2} \quad (3)$$

The equation of heat conduction

$$\rho C_E \left( \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) \theta + \beta_1 T_0 \left( \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) e + a T_0 \left( \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) C = K \theta_{,ii} \quad (4)$$

The equation of mass diffusion

$$D \beta_2 e_{,ii} + D a \theta_{,ii} + \left( \frac{\partial}{\partial t} + \tau \frac{\partial^2}{\partial t^2} \right) C - D b C_{,ii} = 0 \quad (5)$$

where list of symbols are given in the nomenclature.

## 3. Formulation and solution of the problem

We consider an isotropic, homogeneous elastic solid with generalized thermoelastic diffusion in the undeformed state at temperature  $T_0$ . We introduce the rectangular Cartesian coordinate system  $(x, y, z)$  which has its origin on the surface  $z = 0$  with the  $z$ -axis pointing normally into the medium.

For two-dimensional problem, we assume

$$\vec{u} = (u_1, 0, u_3) \quad (6)$$

The initial and regularity conditions are given by

$$\left. \begin{aligned} u_1(x, z, 0) = 0 = \frac{\partial u_1}{\partial t}(x, z, 0) \\ u_3(x, z, 0) = 0 = \frac{\partial u_3}{\partial t}(x, z, 0) \end{aligned} \right\}$$

$$\left. \begin{aligned} \theta(x, z, 0) = 0 = \frac{\partial \theta}{\partial t}(x, z, 0) \\ C(x, z, 0) = 0 = \frac{\partial C}{\partial t}(x, z, 0) \end{aligned} \right\} \text{ for } z \geq 0, -\infty < x < \infty \quad (7)$$

$$u_1(x, z, t) = u_3(x, z, t) = \theta(x, z, t) = C(x, z, t) = 0 \quad \text{for } t > 0 \text{ when } z \rightarrow \infty \quad (8)$$

Assuming time harmonic behavior as

$$(u_1, u_3, \theta, C)(x, z, t) = (u_1, u_3, \theta, C)(x, z)e^{i\omega t} \quad (9)$$

where  $\omega$  is the angular frequency.

To facilitate the solution, the following dimensionless quantities are introduced

$$\begin{aligned} x' &= \frac{\omega_1^*}{c_1}x, & z' &= \frac{\omega_1^*}{c_1}z, & t' &= \omega_1^*t, & u'_1 &= \frac{\omega_1^*}{c_1}u_1 \\ u'_3 &= \frac{\omega_1^*}{c_1}u_3, & t'_{33} &= \frac{t_{33}}{\beta_1 T_0}, & t'_{31} &= \frac{t_{31}}{\beta_1 T_0}, & \omega' &= \frac{\omega}{\omega_1^*} \\ \theta' &= \frac{\beta_1}{\rho c_1^2}\theta, & C' &= \frac{\beta_2}{\rho c_1^2}C, & a' &= \frac{\omega_1^*}{c_1}a, & \tau' &= \omega_1^*\tau \\ \tau'_0 &= \omega_1^*\tau_0, & P' &= \frac{P}{\beta_2}, & P'_1 &= \frac{P_1}{\beta_1 T_0}, & P'_2 &= \frac{\beta_1}{\rho c_1^2}P_2 \\ P'_3 &= \frac{P_3}{\beta_2} \end{aligned} \quad (10)$$

where

$$c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad \omega_1^* = \frac{\rho C_E c_1^2}{K}$$

The displacement components  $u_1(x, z, t)$  and  $u_3(x, z, t)$  may be written in terms of potential functions  $\phi(x, z, t)$  and  $\psi(x, z, t)$  as

$$u_1 = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z}, \quad u_3 = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x} \quad (11)$$

Using Eqs. (6) and (9)–(11), Eqs. (3)–(5) recast into the following form (after suppressing the primes)

$$\left(\nabla^2 + \frac{\omega^2}{\delta}\right)\psi = 0 \quad (12)$$

$$(\nabla^2 + \omega^2)\phi - \theta - C = 0 \quad (13)$$

$$(\nabla^2 - n_1)\theta = \varepsilon_1 n_1 (\nabla^2 \phi + a_1 C) \quad (14)$$

$$\nabla^4 \phi + a_1 \nabla^2 \theta - \varepsilon_2 (\nabla^2 - a_2 n_2)C = 0 \quad (15)$$

where

$$\delta = \frac{\mu}{\lambda + 2\mu}, \quad \varepsilon_1 = \frac{\beta_1^2 T_0}{\rho C_E (\lambda + 2\mu)}, \quad a_1 = \frac{a(\lambda + 2\mu)}{\beta_1 \beta_2}$$

$$\varepsilon_2 = \frac{b(\lambda + 2\mu)}{\beta_2^2}, \quad a_2 = \frac{1}{bD\eta}, \quad \eta = \frac{\rho C_E}{K}$$

$$n_1 = i\omega(1 + \tau_0 i\omega), \quad n_2 = i\omega(1 + \tau i\omega)$$

Applying Fourier transformation defined by

$$\hat{f}(\xi, z, \omega) = \int_{-\infty}^{\infty} f(x, z, \omega)e^{i\xi x} dx \quad (16)$$

on Eqs. (12)–(15), then eliminating  $\hat{\phi}$ ,  $\hat{\theta}$ ,  $\hat{C}$  and  $\hat{\psi}$  from the resulting expression we obtain

$$\left(\frac{d^6}{dz^6} + Q\frac{d^4}{dz^4} + N\frac{d^2}{dz^2} + I\right)(\hat{\phi}, \hat{\theta}, \hat{C}) = 0 \quad (17)$$

$$\left(\frac{d^2}{dz^2} - \lambda_4^2\right)\hat{\psi} = 0 \quad (18)$$

where  $Q, N, I$  and  $\lambda_4$  are listed in Appendix A.

The roots of Eq. (17) are  $\pm\lambda_l$  ( $l = 1, 2, 3$ ) and the roots of Eq. (18) are  $\pm\lambda_4$ . Making use of radiation condition  $\hat{\phi}, \hat{\theta}, \hat{C}$  and  $\hat{\psi} \rightarrow 0$  as  $z \rightarrow \infty$ , the solutions of Eqs. (17) and (18) may be written as

$$\hat{\phi} = A_1 e^{-\lambda_1 z} + A_2 e^{-\lambda_2 z} + A_3 e^{-\lambda_3 z} \quad (19)$$

$$\hat{\theta} = d_1 A_1 e^{-\lambda_1 z} + d_2 A_2 e^{-\lambda_2 z} + d_3 A_3 e^{-\lambda_3 z} \quad (20)$$

$$\hat{C} = e_1 A_1 e^{-\lambda_1 z} + e_2 A_2 e^{-\lambda_2 z} + e_3 A_3 e^{-\lambda_3 z} \quad (21)$$

$$\hat{\psi} = A_4 e^{-\lambda_4 z} \quad (22)$$

with  $A_l$  ( $l = 1, 2, 3$ ) being arbitrary constants and  $d_l$  and  $e_l$  are given in Appendix B.

#### 4. Applications

On the half-space surface ( $z = 0$ ) normal force, thermal source and chemical potential source, which are assumed to be time harmonic, are applied. We consider three types of boundary conditions, as follows:

##### 4.1. The normal force on the surface of half-space

The boundary conditions in this case are

$$\begin{aligned} \text{(i) } t_{33}(x, z, t) &= -P_1 \psi_1(x)e^{i\omega t}, & \text{(ii) } t_{31}(x, z, t) &= 0 \\ \text{(iii) } \theta &= 0, & \text{(iv) } P &= 0, \text{ at } z = 0 \end{aligned} \quad (23)$$

where  $\psi_1(x)$  specify source distribution function along  $x$ -axis and  $P_1$  is the magnitude of force applied.

Making use of Eqs. (1), (2) and (9)–(11) in the boundary conditions (23) and applying the Fourier transform defined by Eq. (16) and substitute the values of  $\hat{\phi}, \hat{\theta}, \hat{C}, \hat{\psi}$  from Eqs. (19)–(22) in the resulting equations, we obtain the expressions for components of displacement, stress, temperature distribution and chemical potential distribution which are given in Appendix C.

##### 4.2. The thermal source on the surface of half-space

When the plane boundary is stress free and subjected to thermal source, the boundary conditions are

$$\begin{aligned} \text{(i) } t_{33}(x, z, t) &= 0, & \text{(ii) } t_{31}(x, z, t) &= 0 \\ \text{(iii) } \theta &= P_2 \psi_1(x)e^{i\omega t}, & \text{(iv) } P &= 0, \text{ at } z = 0 \end{aligned} \quad (24)$$

where  $\psi_1(x)$  is the source distribution function along  $x$ -axis and  $P_2$  is the constant temperature applied on the boundary.

With the help of Eqs. (1), (2) and (9)–(11) and the boundary conditions (24), the corresponding expressions for components

of displacement, stress, temperature distribution and chemical potential distribution are as given by Eqs. (C.1)–(C.6) (Appendix C) with  $\Delta_l$  replaced by  $\Delta_l^*$  ( $l = 1, 2, 3, 4$ ) and  $P_1$  replaced by  $P_2$ , respectively, in Eq. (C.7), where

$$\begin{aligned}\Delta_1^* &= m_1(s_2t_3 - s_3t_2) + s_4(\lambda_2t_3 - \lambda_3t_2) \\ \Delta_2^* &= m_1(s_1t_3 - s_3t_1) + s_4(\lambda_1t_3 - \lambda_3t_1) \\ \Delta_3^* &= m_1(s_1t_2 - s_2t_1) + s_4(\lambda_1t_2 - \lambda_2t_1) \\ \Delta_4^* &= -[s_1(\lambda_2t_3 - \lambda_3t_2) - s_2(\lambda_1t_3 - \lambda_3t_1) + s_3(\lambda_1t_2 - \lambda_2t_1)]\end{aligned}\quad (25)$$

#### 4.3. The chemical potential source on the surface of half-space

Here the boundary is stress free and subjected to chemical potential source, therefore the boundary conditions are

$$\begin{aligned}\text{(i)} \quad t_{33}(x, z, t) &= 0, & \text{(ii)} \quad t_{31}(x, z, t) &= 0 \\ \text{(iii)} \quad \theta &= 0, & \text{(iv)} \quad P &= P_3\psi_1(x)e^{i\omega t}, \text{ at } z = 0\end{aligned}\quad (26)$$

where  $\psi_1(x)$  is the source distribution function along  $x$ -axis and  $P_3$  is the constant potential applied on the boundary.

Adopting the same procedure as in case of mechanical force and thermal source and using the boundary condition (26) the expressions for components of displacement, stress, temperature distribution and chemical potential distribution are as given by Eqs. (C.1)–(C.6) (Appendix C) by replacing  $\Delta_l$  with  $\Delta_l^{**}$  ( $l = 1, 2, 3, 4$ ) and  $P_1$  with  $P_3$ , respectively, in Eq. (C.7), where

$$\begin{aligned}\Delta_1^{**} &= -[m_1(s_2d_3 - s_3d_2) + s_4(\lambda_2d_3 - \lambda_3d_2)] \\ \Delta_2^{**} &= -[m_1(s_1d_3 - s_3d_1) + s_4(\lambda_1d_3 - \lambda_3d_1)] \\ \Delta_3^{**} &= -[m_1(s_1d_2 - s_2d_1) + s_4(\lambda_1d_2 - \lambda_2d_1)] \\ \Delta_4^{**} &= s_1(\lambda_2d_3 - \lambda_3d_2) - s_2(\lambda_1d_3 - \lambda_3d_1) \\ &\quad + s_3(\lambda_1d_2 - \lambda_2d_1)\end{aligned}\quad (27)$$

##### 4.3.1. Case I: Green's functions

To synthesize the Green functions, i.e., the solutions due to concentrated normal force/thermal source chemical potential source on the half-space is obtained by setting

$$\psi_1(x) = \delta(x) \quad (28)$$

in Eqs. (23), (24) and (26). Applying the Fourier transform defined by (16) on Eq. (28) gives

$$\widehat{\psi}_1(\xi) = 1 \quad (29)$$

##### 4.3.2. Case (II): Influence functions

The method to obtain the half-space influence function, i.e., the solutions due to distributed load applied on the half-space is obtained by setting

$$\psi_1(x) = \begin{cases} 1 & \text{if } |x| \leq a \\ 0 & \text{if } |x| > a \end{cases} \quad (30)$$

in Eqs. (23), (24) and (26). The Fourier transform with respect to the pair  $(x, \xi)$  for the case of a uniform strip load of unit amplitude and width  $2a$  applied at origin of the coordinate system

( $x = z = 0$ ) in dimensionless form after suppressing the primes becomes

$$\widehat{\psi}_1(\xi) = \left[ 2 \sin\left(\frac{\xi c_1 a}{\omega_1^*}\right) / \xi \right], \quad \xi \neq 0 \quad (31)$$

The expressions for displacements, stresses, temperature distribution and chemical potential distribution can be obtained for concentrated normal force thermal source/chemical potential source and uniformly distributed normal force/thermal source/chemical potential source by replacing  $\widehat{\psi}_1(\xi)$  from Eqs. (29) and (31), respectively, in Eqs. (C.1)–(C.6) (Appendix C) along with (25) and (27).

## 5. Particular cases

5.1. Neglecting the diffusion effects (i.e.  $\beta_2 = b = a = 0$ ), we obtain the corresponding expressions due to normal force for displacements, stresses, and temperature distribution in generalized thermoelastic half-space which are listed in Eqs. (D.1)–(D.5) (Appendix D).

The above expressions yield the corresponding expressions for concentrated and uniformly distributed normal force by replacing  $\widehat{\psi}_1(\xi)$  from Eqs. (29) and (31) respectively in Eqs. (D.1)–(D.5).

5.2. Making use the values of  $\widehat{\psi}_1(\xi)$  from Eqs. (29) and (31) in Eqs. (D.1)–(D.5) and by replacing  $\Delta_l'$  with  $\Delta_l''$  ( $l = 1, 2, 3$ ) in Eq. (D.6) as given below, we obtain the expressions for displacements, stresses and temperature distribution in thermoelastic medium due to concentrated and uniformly distributed thermal source, where

$$\begin{aligned}\Delta_1'' &= m_1s_2^* + s_4\lambda_2, & \Delta_2'' &= m_1s_1^* + s_4\lambda_1 \\ \Delta_3'' &= s_2^*\lambda_1 - s_1^*\lambda_2\end{aligned}\quad (32)$$

## 6. Special case

In case of coupled thermoelasticity, the relaxation times vanish i.e.  $\tau_0 = \tau = 0$  in equations given in Appendices A and B and consequently, we obtain the corresponding expressions in thermoelastic with diffusion and thermoelasticity, respectively, and changed values are listed in Appendix E.

## 7. Inversion of the transforms

To obtain the solution of the problem in the physical domain, we invert the transforms in Eqs. (C.1)–(C.6) and (D.1)–(D.5), for the two theories, i.e. L–S and C–T theories of thermoelasticity. These expressions are functions of  $z$  and the parameter of Fourier transform  $\xi$ , and hence are of the form  $\widehat{f}(\xi, z)$ . To obtain the function  $f(x, z)$  in the physical domain, we invert the Fourier transform using,

$$\begin{aligned}f(x, z) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\xi x} \widehat{f}(\xi, z) d\xi \\ &= \frac{1}{\pi} \int_0^{\infty} (\cos(\xi x) f_e - i \sin(\xi x) f_0) d\xi\end{aligned}\quad (33)$$

where  $f_e$  and  $f_o$  are, respectively, the even and odd parts of the function  $\hat{f}(\xi, z)$ . The method for evaluating this integral is described by Press et al. [18], which involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

### 8. Numerical results and discussion

Following Sherief and Saleh [11] copper material is chosen for the purpose of numerical calculation.

$$T_0 = 293 \text{ K}, \quad \rho = 8954 \text{ kg m}^{-3}, \quad \tau_0 = 0.02 \text{ s}, \quad \tau = 0.2 \text{ s}$$

$$C_E = 383.1 \text{ J kg}^{-1} \text{ K}^{-1}, \quad \alpha_t = 1.78(10)^{-5} \text{ K}^{-1}$$

$$\alpha_c = 1.98(10)^{-4} \text{ m}^3 \text{ kg}^{-1}, \quad K = 386 \text{ W m}^{-1} \text{ K}^{-1}$$

$$\lambda = 7.76(10)^{10} \text{ kg m}^{-1} \text{ s}^{-2}, \quad \mu = 3.86(10)^{10} \text{ kg m}^{-1} \text{ s}^{-2}$$

$$D = 0.85(10)^{-8} \text{ kg s m}^{-3}, \quad a = 1.2(10)^4 \text{ m}^2 \text{ s}^{-2} \text{ K}^{-1}$$

$$b = 0.9(10)^6 \text{ m}^5 \text{ kg}^{-1} \text{ s}^{-2}$$

The values of normal stress  $t_{33}$ , temperature distribution  $\theta$ , concentration distribution  $C$  and chemical potential distribution  $P$  for thermoelastic diffusion (TED) and thermoelasticity (TE) are studied for normal force/thermal source/chemical potential source. The variations of the components with distance  $x$  are shown (a) solid line for TED and solid line with center symbol 'triangle' for TE for L-S theory, (b) small dashed line for TED and small dashed line with center symbol 'Diamond' for TE for C-T theory. The variations are shown in Figs. 1–12. The computations are carried out for non-dimensional frequency  $\omega = 1.5$  and time  $t = 0.5$  in the range  $0 \leq x \leq 10$ .

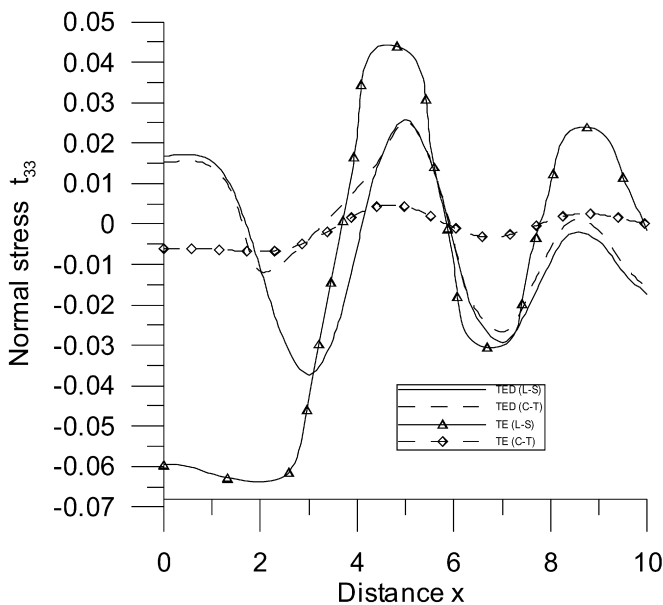


Fig. 1. Variation of normal stress  $t_{33}$  with distance  $x$  (concentrated force).

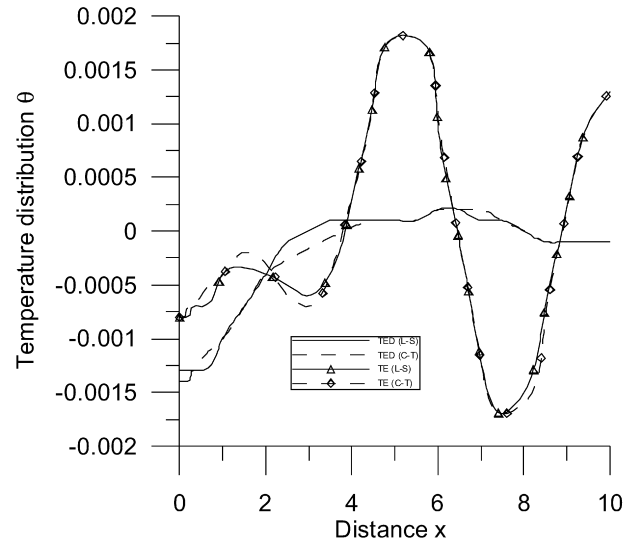


Fig. 2. Variation of temperature distribution  $\theta$  with distance  $x$  (concentrated force).

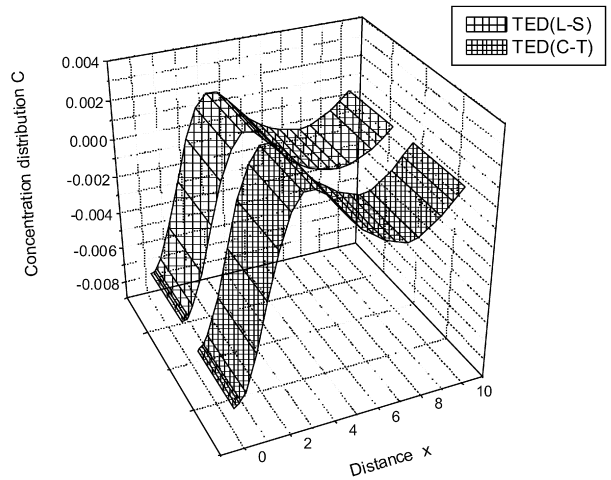


Fig. 3. Variation of concentration distribution  $C$  with distance  $x$  (concentrated force).

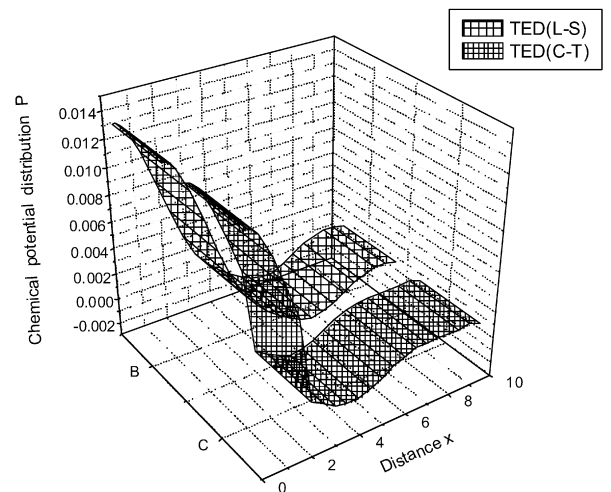


Fig. 4. Variation of chemical potential distribution  $P$  with distance  $x$  (concentrated force).

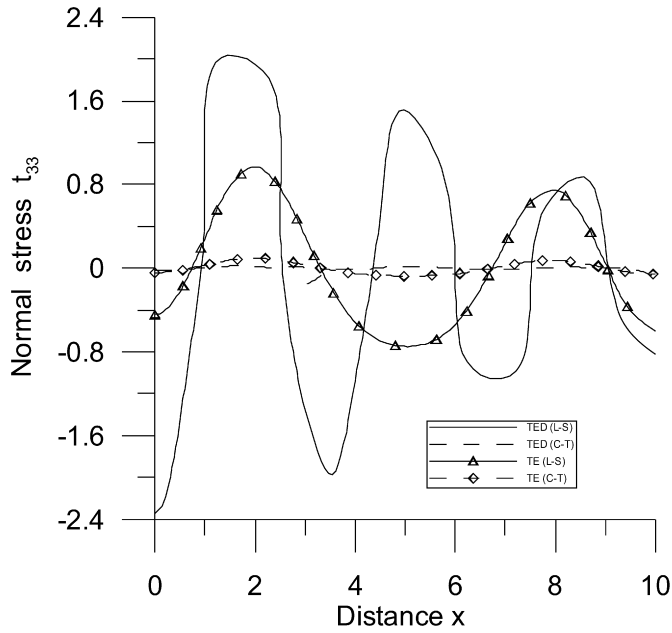


Fig. 5. Variation of normal stress  $t_{33}$  with distance  $x$  (uniformly distributed thermal source).

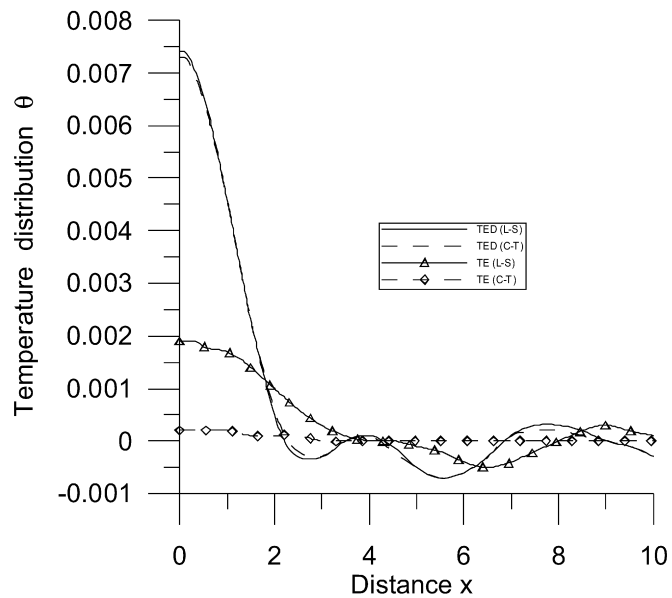


Fig. 6. Variation of temperature distribution  $\theta$  with distance  $x$  (uniformly distributed thermal source).

8.1. Normal force on the surface of half-space

8.1.1. Concentrated force

The variations of normal stress  $t_{33}$  with the distance  $x$  have been shown in Fig.1. The values of normal stress  $t_{33}$  in case of TED (L-S) and TED (C-T) are more than TE (L-S) and TE (C-T) near the loading surface and the distance away from loading surface;  $t_{33}$  follow an oscillatory path with difference in their magnitudes for both theories.

Fig. 2 shows that the values of temperature distribution  $\theta$  have an opposite oscillatory behavior for L-S and C-T theories of thermoelasticity (with diffusion and without diffusion) in the

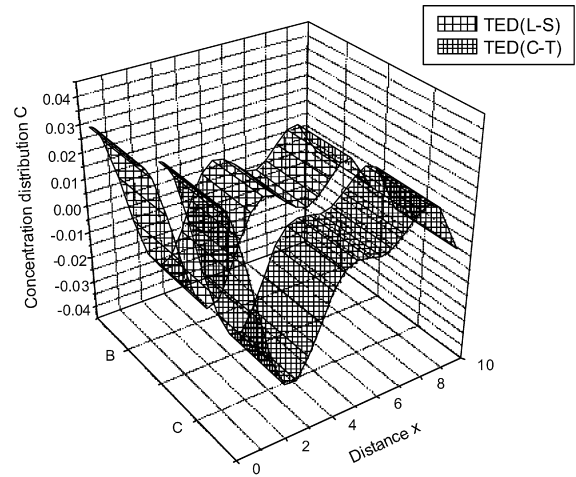


Fig. 7. Variation of concentration distribution  $C$  with distance  $x$  (uniformly distributed thermal source).

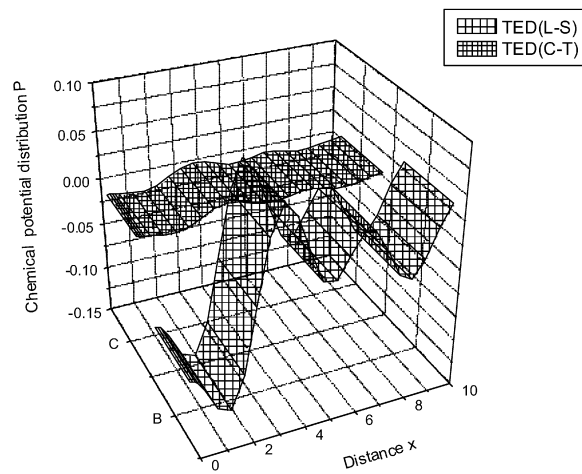


Fig. 8. Variation of chemical potential distribution  $P$  with distance  $x$  (uniformly distributed thermal source).

whole range of  $x$  but magnitude of oscillation in case of TED (L-S) and TED (C-T) is less in comparison to TE (L-S) and TE (C-T).

The values of concentration distribution  $C$  for both the L-S and C-T theories (close to each other) start with sharp increase, with further increase in distance  $x$  the values of  $C$  in case of classical theory of thermoelasticity are more than L-S theory in the intermediate range and smaller in the remaining range. These variations are shown in Fig. 3.

The variations of chemical potential distribution  $P$  with distance  $x$  have been shown in Fig. 4. The values of chemical potential distribution  $P$  for both TED (L-S) and TED (C-T) decreases sharply as  $x$  ranges between  $0 \leq x \leq 6$  and increases as  $x$  increases further.

8.2. Thermal source on the surface of half-space

8.2.1. Uniformly distributed thermal source

The variations of normal stress  $t_{33}$  with the distance  $x$  have been shown in Fig. 5. If we fix the point of observation i.e.

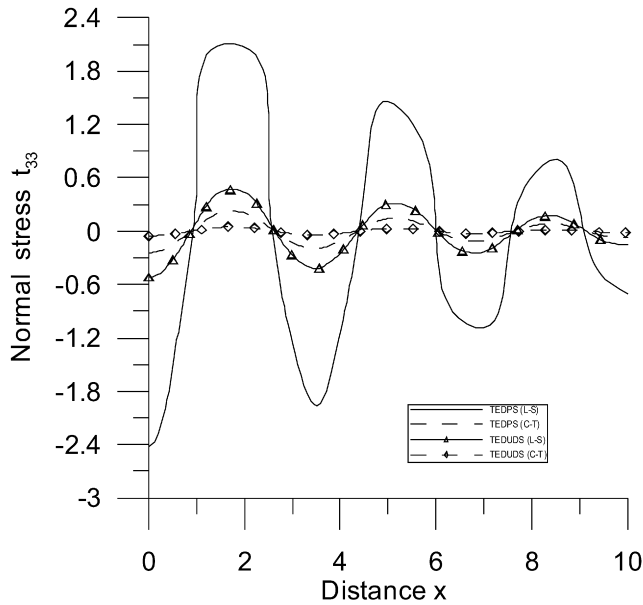


Fig. 9. Variation of normal stress  $t_{33}$  with distance  $x$  (concentrated and uniformly distributed chemical potential sources).

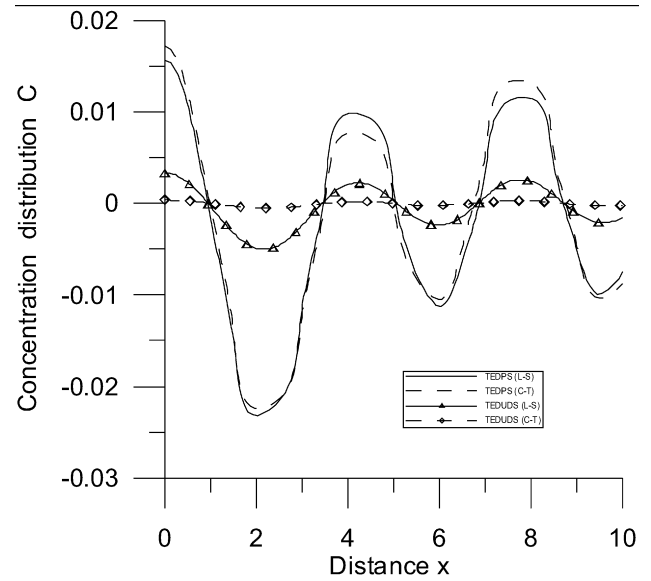


Fig. 11. Variation of concentration distribution  $C$  with distance  $x$  (concentrated and uniformly distributed chemical potential sources).

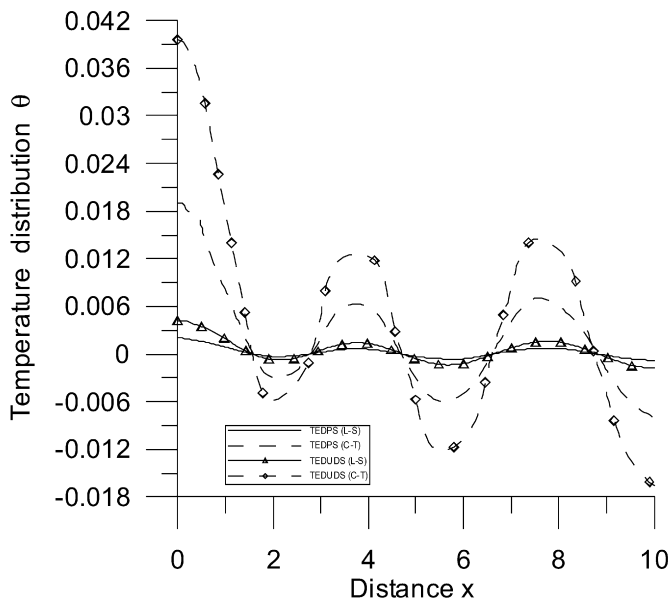


Fig. 10. Variation of temperature distribution  $\theta$  with distance  $x$  (concentrated and uniformly distributed chemical potential sources).

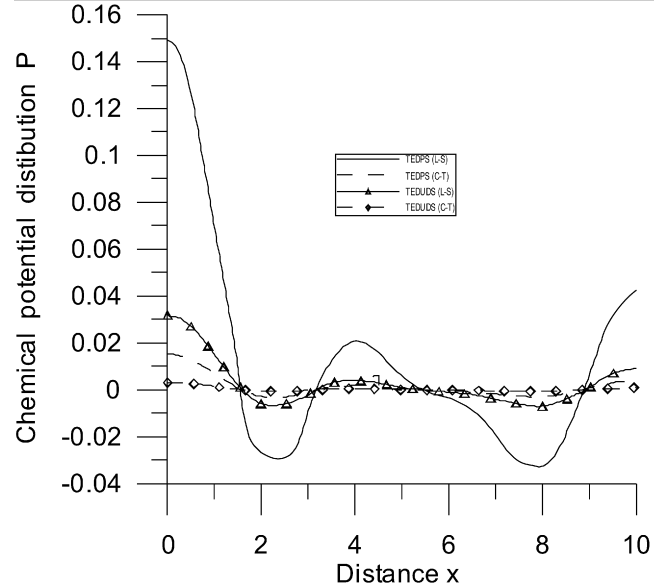


Fig. 12. Variation of chemical potential  $P$  with distance  $x$  (concentrated and uniformly distributed chemical potential sources).

the value of distance  $x$ , the values of normal stress  $t_{33}$  increase and decrease for L-S theory (with and without diffusion) but the oscillatory pattern is not uniform for all the distances  $x$  and for all the values of  $t_{33}$ , depicting the effect of diffusion. The trend of variation of  $t_{33}$  for TED (C-T) and TE (C-T) follow an oscillatory pattern in the whole range of  $x$ .

Due to effect of diffusion, the values of temperature distribution  $\theta$  in case of both TED (L-S) and TED (C-T) are more than TE (L-S) and TE (C-T) in the range  $0 \leq x \leq 1.8$  and with further increase in distance  $x$ , it reveals that all the values of  $\theta$  becomes nearer to zero value for both L-S and C-T theories. These variations are shown in Fig. 6.

It is noticed from Fig. 7 that the trends of variations for concentration distribution  $C$  are almost same for both L-S and C-T theories. The values of concentration distribution  $C$  for both the L-S and C-T theories decrease sharply in the initial range of  $x$ , then follow an oscillatory pattern and gradually decrease to zero value at  $x = 0$ .

The variations of chemical potential distribution  $P$  with distance  $x$  have been shown in Fig. 8. The values of chemical potential distribution  $P$  for classical theory of thermoelasticity is more than for L-S theory near the loading surface and their values oscillates as  $x$  increases further but their magnitude of oscillation are different.

### 8.3. Chemical potential sources on the surface of half-space

- (a) Solid line for thermoelastic diffusion with concentrated source (TEDPS) and solid line with center symbol ‘triangle’ for thermoelastic diffusion with uniformly distributed source (TEDUDS) for L–S theory.
- (b) Small dashed line for thermoelasticity with concentrated source (TEDPS) and small dashed line with center symbol ‘diamond’ for thermoelasticity with uniformly distributed source (TEDUDS) for C–T theory.

#### 8.3.1. Concentrated and uniformly distributed chemical potential sources

The variations of normal stress  $t_{33}$  with the distance  $x$  have been shown in Fig. 9. The values of  $t_{33}$  in case of concentrated source as well as uniformly distributed source show an oscillatory behavior in the whole range of  $x$  for both the theories of thermoelasticity (L–S and C–T) but the magnitude of oscillation for uniformly distributed source are lesser than in comparison to concentrated source.

For both concentrated source and uniformly distributed source, the values of temperature distribution  $\theta$  in case of C–T theory are more than L–S theory at the loading surface and the trend of variations of  $\theta$  is oscillatory in the remaining range for both the theories with difference in their magnitudes. These variations are shown in Fig. 10.

It is seen from Fig. 11, that the type of source applied affect only on the magnitude and not on the trend of the curves. The values of concentration distribution  $C$  follow an oscillatory pattern in the whole range, but, in case of concentrated source the magnitude of oscillation is more in comparison to uniformly distributed source for both the theories of thermoelasticity (L–S and C–T).

The values of chemical potential distribution  $P$  in case of concentrated source are greater than uniformly distributed source for both classical theory (C–T) and L–S theory in the initial range of  $x$  and then values of  $P$  are close to each other in the remaining range except for TED (L–S). These variations are shown in Fig. 12.

## 9. Conclusion

Frequency and effect of diffusion play an important role in the study of the deformation of the body. Near the application of the source, diffusivity increases the values of normal stress and decreases the values of temperature distribution for concentrated force while for uniformly distributed thermal source, diffusivity decreases the values of normal stress and increases the values of temperature distribution and the behavior of variations are oscillatory for both sources as distance away from the loading surface. The values of normal stress, temperature distribution and concentration distribution in case of chemical potential sources for C–T theory are more than L–S theory near the point of application of source and follow an oscillatory pattern away from the source. As the disturbances travels through different constituents of the medium, it suffers sudden changes, resulting in an inconsistent/non-uniform pattern of curves. The

trend of curves exhibits the properties of thermo-diffusivity of the medium and satisfies the requisite condition of the problem. The results of this problem are very useful in the two dimensional problem of dynamic response due to various sources of the thermoelastic diffusion which has the various geophysical and industrial application.

## Appendix A

$$Q = \frac{1}{A} [B - 3\xi^2 A]$$

$$N = \frac{1}{A} [C - 2B\xi^2 + 3\xi^4 A]$$

$$I = \frac{1}{A} [B\xi^4 - C\xi^2 + D - A\xi^6]$$

$$\lambda_4^2 = \xi^2 - \frac{\omega^2}{\delta}$$

and

$$A = -\varepsilon_2 + 1$$

$$B = \varepsilon_2(n_2 a_2 + n_1 - \omega^2 + \varepsilon_1 n_1) + \varepsilon_1 a_1 n_1 (a_1 + 2) - n_1$$

$$C = \varepsilon_2 a_2 \left( n_1 n_2 + \omega^2 n_2 + \frac{\omega^2}{a_2} n_1 - \varepsilon_1 n_1 n_2 \right) + \omega^2 \varepsilon_1 a_1^2 n_1$$

$$D = \omega^2 \varepsilon_2 a_2 n_1 n_2$$

## Appendix B

$$d_l = \frac{P^* \lambda_l^2 + Q^*}{R^* \lambda_l^2 + S^*}, \quad e_l = \frac{U^* \lambda_l^4 + V^* \lambda_l^2 + W^*}{X^* \lambda_l^2 + T^*} \quad (l = 1, 2, 3)$$

$$P^* = \frac{a_1 + 1}{a_1}, \quad U^* = 1 + a_1$$

$$Q^* = -\left[ (\xi^2 - \omega^2) + \frac{\xi^2}{a_1} \right], \quad V^* = -(2(1 + a_1)\xi^2 - a_1 \omega^2)$$

$$R^* = \frac{1}{\varepsilon_1 a_1 n_1}, \quad W^* = (1 + a_1)\xi^4 - a_1 \omega^2 \xi^2$$

$$S^* = 1 - \frac{1}{\varepsilon_1 a_1 n_1} (\xi^2 + n_1), \quad X^* = a_1 + \varepsilon_2$$

$$T^* = -\{ [a_1 + \varepsilon_2]\xi^2 + \varepsilon_2 a_2 n_2 \}$$

## Appendix C

The expressions for components of displacement, stress, temperature distribution and chemical potential distribution are given as

$$\hat{u}_1 = \frac{1}{\Delta} \{ P_1 \hat{\psi}_1(\xi) [(-i\xi)(\Delta_1 e^{-\lambda_1 z} - \Delta_2 e^{-\lambda_2 z} + \Delta_3 e^{-\lambda_3 z}) + \lambda_4 \Delta_4 e^{-\lambda_4 z}] \} e^{i\omega t} \quad (C.1)$$

$$\hat{u}_3 = \frac{-1}{\Delta} \{ P_1 \hat{\psi}_1(\xi) [\lambda_1 \Delta_1 e^{-\lambda_1 z} - \lambda_2 \Delta_2 e^{-\lambda_2 z} + \lambda_3 \Delta_3 e^{-\lambda_3 z} + i\xi \Delta_4 e^{-\lambda_4 z}] \} e^{i\omega t} \quad (C.2)$$

$$\hat{t}_{33} = \frac{1}{\Delta} \{ P_1 \hat{\psi}_1(\xi) [s_1 \Delta_1 e^{-\lambda_1 z} - s_2 \Delta_2 e^{-\lambda_2 z} + s_3 \Delta_3 e^{-\lambda_3 z} + s_4 \Delta_4 e^{-\lambda_4 z}] \} e^{i\omega t} \quad (C.3)$$



$$t_{31} = \frac{1}{\Delta} \{ P_1 \widehat{\psi}_1(\xi) [\lambda_1 \Delta_1 e^{-\lambda_1 z} - \lambda_2 \Delta_2 e^{-\lambda_2 z} + \lambda_3 \Delta_3 e^{-\lambda_3 z} - m_1 \Delta_4 e^{-\lambda_4 z}] \} e^{i\omega t} \tag{C.4}$$

$$\hat{\theta} = \frac{1}{\Delta} \{ P_1 \widehat{\psi}_1(\xi) [d_1 \Delta_1 e^{-\lambda_1 z} - d_2 \Delta_2 e^{-\lambda_2 z} + d_3 \Delta_3 e^{-\lambda_3 z}] \} e^{i\omega t} \tag{C.5}$$

$$\hat{P} = \frac{1}{\Delta} \{ P_1 \widehat{\psi}_1(\xi) [t_1 \Delta_1 e^{-\lambda_1 z} - t_2 \Delta_2 e^{-\lambda_2 z} + t_3 \Delta_3 e^{-\lambda_3 z}] \} e^{i\omega t} \tag{C.6}$$

where

$$\begin{aligned} \Delta &= \{ -[s_4 \lambda_1 + m_1 s_1] (d_2 t_3 - d_3 t_2) + [s_4 \lambda_2 + m_1 s_2] \\ &\quad \times (d_1 t_3 - d_3 t_1) - [s_4 \lambda_3 + m_1 s_3] (d_1 t_2 - d_2 t_1) \} \\ \Delta_1 &= m_1 (d_2 t_3 - d_3 t_2), \quad \Delta_2 = m_1 (d_1 t_3 - d_3 t_1), \\ \Delta_3 &= m_1 (d_1 t_2 - d_2 t_1) \\ \Delta_4 &= [\lambda_1 (d_2 t_3 - d_3 t_2) - \lambda_2 (d_1 t_3 - d_3 t_1) + \lambda_3 (d_1 t_2 - d_2 t_1)] \\ s_l &= b_1 \lambda_l^2 - b_1 d_l - b_1 e_l - b_2 i \xi \quad (l = 1, 2, 3) \\ s_4 &= (i \xi b_1 + b_2) \lambda_4 \\ t_l &= \xi^2 - \lambda_l^2 - \varepsilon_2 e_l - a_1 d_l \end{aligned} \tag{C.7}$$

### Appendix D

The expressions for components of displacement, stress, temperature distribution in thermoelastic half-space are given as

$$\hat{u}_1 = \frac{1}{\Delta^*} \{ P_1 \widehat{\psi}_1(\xi) [(-i \xi) (-\Delta'_1 e^{-\lambda_1 z} + \Delta'_2 e^{-\lambda_2 z}) - \lambda_4 \Delta'_3 e^{-\lambda_4 z}] \} e^{i\omega t} \tag{D.1}$$

$$\hat{u}_3 = \frac{-1}{\Delta^*} \{ P_1 \widehat{\psi}_1(\xi) [-\lambda_1 \Delta'_1 e^{-\lambda_1 z} + \lambda_2 \Delta'_2 e^{-\lambda_2 z} - i \xi \Delta'_3 e^{-\lambda_4 z}] \} e^{i\omega t} \tag{D.2}$$

$$\hat{t}_{33} = \frac{1}{\Delta^*} \{ P_1 \widehat{\psi}_1(\xi) [-s_1^* \Delta'_1 e^{-\lambda_1 z} + s_2^* \Delta'_2 e^{-\lambda_2 z} - s_4^* \Delta'_3 e^{-\lambda_4 z}] \} e^{i\omega t} \tag{D.3}$$

$$t_{31} = \frac{1}{\Delta^*} \{ P_1 \widehat{\psi}_1(\xi) [-\lambda_1 \Delta'_1 e^{-\lambda_1 z} + \lambda_2 \Delta'_2 e^{-\lambda_2 z} + m_1 \Delta'_3 e^{-\lambda_4 z}] \} e^{i\omega t} \tag{D.4}$$

$$\hat{\theta} = \frac{1}{\Delta^*} \{ P_1 \widehat{\psi}_1(\xi) [-d_1 \Delta'_1 e^{-\lambda_1 z} + d_2 \Delta'_2 e^{-\lambda_2 z}] \} e^{i\omega t} \tag{D.5}$$

where

$$\begin{aligned} \Delta^* &= [s_4 (\lambda_1 d_2 - \lambda_2 d_1) + m_1 (s_1^* d_2 - s_2^* d_1)] \\ \Delta'_1 &= m_1 d_2, \quad \Delta'_2 = m_1 d_1, \quad \Delta'_3 = [\lambda_1 d_2 - \lambda_2 d_1] \\ s_l^* &= b_1 \lambda_l^2 - b_1 d_l - b_2 i \xi, \quad (l = 1, 2) \\ s_4 &= (i \xi b_1 + b_2) \lambda_4 \end{aligned} \tag{D.6}$$

### Appendix E

$$B = i\omega \varepsilon_2 (a_2 + 1 + i\omega + \varepsilon_1) + \varepsilon_1 a_1 i\omega (a_1 + 2) - i\omega$$

$$C = \varepsilon_2 a_2 \omega^2 \left( 1 + i\omega + \frac{i\omega}{a_2} + \varepsilon_1 \right) + i\omega^3 \varepsilon_1 a_1^2$$

$$D = \omega^4 \varepsilon_2 a_2$$

$$R^* = \frac{1}{\varepsilon_1 a_1 i\omega}, \quad S^* = 1 - \frac{\xi^2 + i\omega}{\varepsilon_1 a_1 i\omega}$$

$$T^* = -\{ [a_1 + \varepsilon_2] \xi^2 + i\omega \varepsilon_2 a_2 \}$$

$$s_l^* = b_1 \lambda_l^2 - b_1 d_l - b_2 i \xi \quad (l = 1, 2)$$

$$s_l = b_1 \lambda_l^2 - b_1 d_l - b_1 e_l - b_2 i \xi \quad (l = 1, 2, 3)$$

$$s_4 = (i \xi b_1 + b_2) \lambda_4$$

$$t_l = \xi^2 - \lambda_l^2 - \varepsilon_2 e_l - a_1 d_l$$

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